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Entropy function and attractors for AdS black holes

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Abstract: We apply Sen's entropy formalism to the study of the near horizon geometry and the entropy of asymptotically AdS black holes in gauged supergravities. In particular, we consider non-supersymmetric electrically charged black holes with $AdS_2 \times S^{d-2}$ horizons in $U(1)^4$ and $U(1)^3$ gauged supergravities in $d = 4$ and $d = 5$ dimensions, respectively. We study several cases including static/rotating, BPS and non-BPS black holes in Einstein as well as in Gauss-Bonnet gravity. In all examples, the near horizon geometry and black hole entropy are derived by extremizing the entropy function and are given entirely in terms of the gauge coupling, the electric charges and the angular momentum of the black hole.

Keywords: AdS-CFT Correspondence, Black Holes in String Theory.

Contents

1. Introduction

The study of black hole thermodynamics has played a central role in the development of our current notions of holography in gravity. In this line of thinking, black holes are viewed as thermodynamic objects at equilibrium with a temperature and an entropy. A simple analysis of this thermodynamic system leads to the remarkable Bekenstein-Hawking formula for the black hole entropy. This formula relates the entropy of the black hole to the area of its horizon and it suggests that the microscopic degrees of freedom of the black hole can be described by a "dual" quantum mechanics living on the horizon. This is further supported by the discovery of AdS/CFT dualities [1] that relate gravity on AdS spaces and gauge theories living on the AdS boundary. These observations drastically simplify the study of black hole physics, since the geometry of the horizon is typically much simpler than that of the full solution. Even in theories with scalar fields and a large number of moduli – asymptotic values of massless scalars at infinity –, scalars are attracted at the black hole horizon to special values and the full geometry is entirely determined in terms of the black hole charges. This is referred as the *attractor mechanism* [2 – 5]. Originally discussed in the context of $\mathcal{N} = 2$ black holes the attractor mechanism has been recently extended in many directions, including non-supersymmetric and higher derivative gravity theories [6 – 19]. The results show that the attractor mechanism is a universal issue of any gravity theory.

In [20], A. Sen introduced a unifying formalism, the *entropy formalism*, that describes the attractor equations and black hole entropy in a general non-supersymmetric and higher derivative gravity theory. In this formalism, the near horizon geometry is determined by extremizing a single function F, the *entropy function*. The entropy of the black hole is given by the value of F at the extremum. The function F is defined by the Legendre transform with respect to the black hole charges of the gravity action evaluated at the horizon. More precisely, the gravity action is first evaluated at a trial background geometry with volumes and scalar/gauge field profiles parametrized by a finite number of parameters. These parameters are then determined by extremizing the entropy function F. The formalism has been successfully applied to the study of general non-supersymmetric asymptotically flat black holes in various supergravity settings [21 − 32].

The aim of this paper is to extend this analysis to the study of asymptotically AdS black holes in gauged supergravities. According to holography [1] the entropy of black holes in AdS spaces is related to the free energy of the dual gauge theory living on the AdS boundary, see [33 − 38]. To pursue the study of these holographic correspondences a detailed knowledge of the black hole near horizon data is required. To derive explicit formulas for the attractor geometry and for the entropy of AdS black holes is one of the main motivations of the present work.

Black holes in gauged supergravities are different from those in Poincaré supergravities in many respects. First, in the gauged theory the asymptotic values of the scalar fields at infinity are typically fixed at the minimum of a scalar potential. The moduli space is therefore reduced and often empty. Still once charges are placed on AdS_d, even scalars fixed at infinity flow at the horizon to a different fixpoint specified completely by the black hole charges. I.e. the attractor mechanism now describes a flow between two fixpoint geometries. Second, it is well known that asymptotically AdS black hole solutions with regular horizons are always non-supersymmetric unless a non-trivial angular momentum is turned on. This is very different from the Minkowski case where BPS static solutions are quite common. Our analysis here explores both non-supersymmetric static and rotating black hole solutions.

We apply the entropy formalism to non-supersymmetric black holes with near horizon geometry $AdS_2 \times S^{d-2}$ in $d = 4, 5$[1]. Black holes with these type of horizons have always zero temparature (with coinciding inner and outer horizons) but they are in general non-supersymmetric. For concreteness we focus on the $U(1)^4$ and $U(1)^3$ gauged supergravities in $d = 4$ and $d = 5$, respectively. These theories can be embedded into the maximal gauged supergravities with gauge groups $SO(8)$ and $SO(6)$, respectively, following from compactifications of M-theory and type IIB theory on $AdS_4 \times S^7$ and $AdS_5 \times S^5$, respectively. Black holes in these gauged supergravities have been extensively studied and classified in full generality in the literature [39 − 50] (see [51] for a review and a list of references). In the case of Einstein gravity, the solutions derived here via the entropy formalism follow from these general solutions by taking the zero temperature limit. Our focus here is on the near horizon geometry and black hole entropy.

[1] More precisely, in the case of rotating black holes the horizons are described by a "squashed $AdS_2 \times S^{d-2}$" rather than a tensor product geometry.

We test the entropy formalism in a number of examples, including static/rotating black holes with or without supersymmetry in Einstein as well as Gauss-Bonnet gravity. In each case we show that the attractor geometry follows from extremization of the entropy function. In the case of Einstein gravity the entropy function output will be shown in agreement with the Bekenstein-Hawking formula as expected.

The entropy formalism is particularly efficient in the study of black holes in higher derivative gravity. Higher derivative corrections to black hole entropies in rigid supergravities were first studied in [52 − 55]. Higher derivative corrections to asymptotically AdS black holes in Gauss-Bonnet gravity were studied in [56]. More recently in [57] the authors consider several examples of higher derivative terms and derive the first corrections to the Schwarzschild AdS black holes. Here we consider the Einstein-Maxwell system in the presence of a Gauss-Bonnet term and derive exact expressions for the near horizon geometry and the black hole entropy.

The paper is organized as follows: In sections 2 and 3 we consider non-rotating asymptotically AdS black holes in $U(1)^4$ and $U(1)^3$ gauged supergravities in $d = 4$ and $d = 5$, respectively. In section 4 we apply the entropy formalism to rotating black holes in $d = 5$ gauged supergravity. The study of higher derivative corrections is sketched in section 5 for the Gauss-Bonnet type of interactions in the Maxwell-Einstein system in $d = 4, 5$ dimensions. In Section 6 we summarize our results and draw some conclusions. Appendix A contains a discussion on the normalization of the physical charges used in the main text. Appendix B presents the link between our $AdS_2 \times S^{d-2}$ solutions and zero temperature limits of the general black hole solutions.

2. AdS$_4$ static black holes

We start by considering $U(1)^4$ gauged supergravity in four dimensions. This theory follows from a truncation of the maximal $N = 8$, $SO(8)$ gauged supergravity [58] down to the Cartan subgroup of $SO(8)$. The bosonic action can be written as [41]:

$$S = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \left[R - \tfrac{1}{4} X_I^2 F^I_{\mu\nu} F^{\mu\nu I} - \tfrac{1}{2} X_I^{-2} \partial_\mu X_I \partial^\mu X_I - V \right] , \tag{2.1}$$

with $I = 1, \dots, 4$, and

$$F^I_{\mu\nu} = 2\partial_{[\mu} A^I_{\nu]} , \qquad V = -4 g^2 \sum_{I<J} X_I X_J , \qquad X_1 X_2 X_3 X_4 = 1 . \tag{2.2}$$

The equations of motion derived from this lagrangian are:

$$\begin{aligned}
&R_{\mu\nu} - \tfrac{1}{2} X_I^2 F^I_{\mu\sigma} F^{I\sigma}_\nu - \tfrac{1}{2} X_I^{-2} \partial_\mu X_I \partial_\nu X_J - \tfrac{1}{2} g_{\mu\nu} \left(R - \tfrac{1}{4} X_I^2 F^{I2} - \tfrac{1}{2} (X_I^{-1} \partial X_I)^2 - V \right) = 0 , \\
&\frac{\delta}{\delta X_I} \left(\tfrac{1}{4} X_I^2 F^{I2} + \tfrac{1}{2} (X_I^{-1} \partial X_I)^2 + V \right) = 0 , \\
&\partial_\mu (\sqrt{-g}\, X_I^2 F^{\mu\nu I}) = 0 .
\end{aligned} \tag{2.3}$$

We look for non-rotating black hole solutions with $AdS_2 \times S^2$ near horizon geometry

$$\begin{aligned}
ds^2 &= v_1 \Big(-r^2\, dt^2 + \frac{dr^2}{r^2}\Big) + v_2\, d\Omega_2 \;, \\
X_I &= u_I \;, \qquad A^I = -e_I\, r\, dt \;, \qquad F^I_{0r} = e_I \;, \\
d\Omega_2 &= (d\theta^2 + \sin^2\theta\, d\phi^2) \qquad 0 \le \theta \le \pi \quad 0 \le \phi < 2\pi \;,
\end{aligned} \tag{2.4}$$

with constants u_I, e_I, v_a, and $u_4 = 1/(u_1 u_2 u_3)$.

The attractor equations determining the constants u_I, v_a, e_I at the black hole horizon are efficiently described by the so called entropy formalism [20]. One starts by evaluating the supergravity action (integrated on the S^2 horizon) in the background (2.4):

$$f(\vec{e},\vec{v},\vec{u}) \equiv \int d\theta d\phi\, \sqrt{-g}\, \mathcal{L}(\vec{e},\vec{v},\vec{u}) \;, \tag{2.5}$$

with $\mathcal{L}(\vec{e},\vec{v},\vec{u})$ the Lagrangian density evaluated on the ansatz (2.4). The *entropy function* $F(\vec{q},\vec{e},\vec{v},\vec{u})$ is then defined as the Legendre transform of f with respect to the charges e_I, i.e.

$$\begin{aligned}
F(\vec{q},\vec{e},\vec{v},\vec{u}) &\equiv 2\pi \Big[e_I q^I - f(\vec{e},\vec{v},\vec{u}) \Big] \\
&= 2\pi \Big[e_I q^I - \frac{v_1 v_2}{4G_4} \Big(-\frac{2}{v_1} + \frac{2}{v_2} + \sum_{I=1}^{4} \frac{u_I^2 e_I^2}{2 v_1^2} + 4g^2 \sum_{I<J}^{4} u_I u_J \Big) \Big] \;.
\end{aligned} \tag{2.6}$$

The near horizon geometry can be found by extremizing the entropy function $F(\vec{q},\vec{e},\vec{v},\vec{u})$ wity respect to $\vec{e},\vec{v}$, and $\vec{u}$:

$$\frac{\partial F}{\partial v_a} = \frac{\partial F}{\partial u_I} = \frac{\partial F}{\partial e_I} = 0 \;. \tag{2.7}$$

The first two equations ensure that the metric and the scalar field equations of motion are satisfied, while the last equation defines the black hole electric charges q^I

$$q^I = \frac{\delta}{\delta e_I} f(\vec{e},\vec{v},\vec{u}) = \frac{v_2}{4\, G_4\, v_1}\, u_I^2 e_I = -\frac{1}{16\pi G_4} \int_{S^2} X_I^2 * F^I \;. \tag{2.8}$$

In the following we will take $G_4 = \frac{1}{8}$ in such a way that the charges q^I are normalized to be integers. This normalization is determined in Appendix A by matching the physical charge units here with those coming from string theory brane setups. The G_4 dependence can be restored by the rescaling (A.6) of the physical charges q^I.

Evaluating the entropy function F at the extremum $(\vec{e}_0(\vec{q}), \vec{v}_0(\vec{q}), \vec{u}_0(\vec{q}))$ one finds the entropy of the corresponding black hole solution as a function of the electric charges $\vec{q}$:

$$S_{\rm BH}(\vec{q}) = F(\vec{q}, \vec{e}_0(\vec{q}), \vec{v}_0(\vec{q}), \vec{u}_0(\vec{q})) \;. \tag{2.9}$$

In practice, the relations (2.8) are highly nonlinear and generically hard to invert, therefore we will often choose to give an implicit parametrization of the black hole solution, its entropy, and the electric charges q^I in terms of $u_{1,2,3}$ and v_2 rather than expressing the entropy directly in terms of the four physical charges q^I.

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It is important to stress that the entropy function formalism applies to (in general non-supersymmetric) higher derivative Lagrangians that depend only on the Riemann and the stress energy tensor but not on their covariant derivatives. In this section we consider Einstein gravity, while higher derivative corrections to black hole entropies will be considered in section 5.

2.1 The solution

As we mentioned in our preliminary discussion, it is often easier to solve equations (2.7), 2.8 implicitly in terms of a set of independent parameters rather than in terms of the four charges q^I. We choose parameters μ_I to parametrize the fixed value scalars $u_{1,2,3}$ and the sphere volume v_2:

$$u_I = \frac{\mu_I}{(\mu_1\mu_2\mu_3\mu_4)^{1/4}} , \qquad v_2 = \tfrac{1}{4}\sqrt{\mu_1\mu_2\mu_3\mu_4} , \tag{2.10}$$

Plugging (2.10) into (2.7) and solving for the remaining variables, one finds the general solution:

$$\begin{aligned} v_1 &= \frac{\frac{1}{4}\sqrt{\mu_1\mu_2\mu_3\mu_4}}{1+g^2\sum_{J<K}\mu_J\mu_K} , \qquad e_I = \frac{\sqrt{\mu_1\mu_2\mu_3\mu_4\,(1+g^2\sum_{I\neq J<K\neq I}\mu_J\mu_K)}}{2\mu_I\,(1+g^2\sum_{J<K}\mu_J\mu_K)} , \\ q^I &= \mu_I\sqrt{1+g^2\sum_{I\neq J<K\neq I}\mu_J\mu_K} . \end{aligned} \tag{2.11}$$

It is easy to check that the equations of motion (2.3) are satisfied by (2.4), (2.10), (2.11). Plugging this into the entropy function (2.6) yields for the black hole entropy

$$S_{\mathrm{BH}}(q) = 2\pi\sqrt{\mu_1\mu_2\mu_3\mu_4} = \frac{\pi v_2}{G_4} = \frac{1}{4G_4}\mathcal{A}_{\mathrm{hor}} , \tag{2.12}$$

in agreement with the Bekenstein-Hawking formula.

In order to express the entropy directly in terms of the electric charges q^I, one has to invert the last equation in (2.11). In lowest orders of the gauge coupling this gives rise to the expansion

$$\mu_I = q^I\left(1-\tfrac{1}{2}g^2\,\partial_I\beta_3+\tfrac{1}{8}g^4\left(\partial_I\,(3\beta_2\beta_3+\beta_1\beta_4)-2q^I\,\partial_I^2\,(\beta_2\beta_3)+4\beta_4\right)+\ldots\right) , \tag{2.13}$$

in terms of the symmetric polynomials

$$\beta_1 = \textstyle\sum_I q^I , \quad \beta_2 = \sum_{I<J} q^I q^J , \quad \beta_3 = \sum_{I<J<K} q^I q^J q^K , \quad \beta_4 = q_1q_2q_3q_4 ,$$

and with $\partial_I = \frac{\partial}{\partial q^I}$. For the entropy this leads to the expansion

$$\begin{aligned} S_{\mathrm{BH}} = 2\pi\sqrt{\beta_4}\,\Big(1&-\tfrac{1}{2}\,g^2\,\beta_2+\tfrac{1}{8}\,g^4\,(3\beta_2^2+2\beta_1\beta_3+4\beta_4) \\ &-\tfrac{1}{16}\,g^6\,(5\beta_2^3+9\beta_1\beta_2\beta_3+\beta_3^2+5\beta_1^2\beta_4+20\beta_2\beta_4)+\ldots\Big) . \end{aligned} \tag{2.14}$$

The expansion drastically simplifies in two particular cases:

Ungauged theory. At $g = 0$ one finds $q^I = \mu_I$ leading to:

$$v_1 = v_2 = \tfrac{1}{4}\sqrt{q_1 q_2 q_3 q_4}\,, \qquad u_I = \frac{q^I}{(q_1 q_2 q_3 q_4)^{1/4}}\,, \qquad e_I = \frac{1}{2q^I}\sqrt{q_1 q_2 q_3 q_4}\,, \tag{2.15}$$

and one recovers the known result

$$S_{\rm BH}(\vec{q}) = 2\pi\,\sqrt{q_1 q_2 q_3 q_4}\,, \tag{2.16}$$

for the entropy in terms of the physical charges.

Equal charges $q^I = q$. In the case of equal charges, the last equation in 2.11 can be explicitly solved for μ and one obtains the explicit solution

$$\begin{aligned} v_1 &= \frac{\sqrt{1+12g^2q^2}-1}{24g^2\sqrt{1+12g^2q^2}}\,, \qquad v_2 = \frac{\sqrt{1+12g^2q^2}-1}{24g^2}\,, \qquad u_I = 1\,, \\ e_I &= \frac{q}{2\sqrt{1+12g^2q^2}}\,, \end{aligned} \tag{2.17}$$

and the black hole entropy

$$S_{\rm BH}(q) = \frac{\pi\,(\sqrt{1+12\,q^2\,g^2}-1)}{3\,g^2}\,, \tag{2.18}$$

expressed directly in terms of the electric charges q.

3. AdS$_5$ static black holes

Next we consider the $U(1)^3$ gauged supergravity in $d = 5$ dimensions. This theory can be obtained as a truncation of the maximal $N = 8$, $SO(6)$ gauged supergravity [59] down to the $U(1)^3$ Cartan subgroup of $SO(6)$. The bosonic action can be written as

$$\begin{aligned} S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\,\Big[R &- \tfrac{1}{4}X_I^2 F^I_{\mu\nu}F^{\mu\nu I} - \tfrac{1}{2}\,X_I^{-2}\,\partial_\mu X_I\,\partial^\mu X_I - V \\ &+ \tfrac{1}{24}\omega^{\mu\nu\sigma\rho\lambda}\,|\epsilon_{IJK}|\,F^I_{\mu\nu}\,F^J_{\sigma\rho}\,A^K_\lambda\Big]\,, \end{aligned} \tag{3.1}$$

with $I = 1, 2, 3$, $\omega^{tr\psi\theta\phi} = -(\sqrt{-g})^{-1}$, and

$$F^I_{\mu\nu} = 2\partial_{[\mu}A^I_{\nu]}\,, \qquad V = -4\,g^2\sum_{I=1}^{3}X_I\,, \qquad X_1X_2X_3 = 1\,. \tag{3.2}$$

The equations of motion derived from this Lagrangian are:

$$\begin{aligned} &R_{\mu\nu} - \tfrac{1}{2}\,X_I^2\,F^I_{\mu\sigma}\,F^{I\sigma}_{\nu} - \tfrac{1}{2}X_I^{-2}\partial_\mu X_I\partial_\nu X_I - \tfrac{1}{2}g_{\mu\nu}\left(R - \tfrac{1}{4}X_I^2F^{I2} - \tfrac{1}{2}(X_I^{-1}\partial X_I)^2 - V\right) = 0\,, \\ &\frac{\delta}{\delta X_I}(\tfrac{1}{4}X_I^2F^{I2} + \tfrac{1}{2}\,(X_I^{-1}\partial X_I)^2 + V) = 0\,, \\ &\frac{1}{\sqrt{-g}}\,\partial_\mu(\sqrt{-g}\,X_I^2\,F^{\mu\lambda I}) + \tfrac{1}{8}\,|\epsilon_{IJK}|\,\omega^{\mu\nu\sigma\rho\lambda}\,F^J_{\mu\nu}\,F^K_{\sigma\rho} = 0\,. \end{aligned} \tag{3.3}$$

We search for non-rotating black holes with near horizon $AdS_2 \times S^3$ geometries

$$
\begin{aligned}
ds^2 &= v_1\Big(-r^2\,dt^2 + \frac{dr^2}{r^2}\Big) + v_2 d\Omega_3 \ , \\
X_I &= u_I \ , \qquad A^I \ = \ -\, e_I\, r\, dt \ , \qquad F^I_{0r} = e_I \ , \\
d\Omega_3 &= \tfrac{1}{4}\left[d\theta^2 + d\psi^2 + d\phi^2 + 2 d\phi\, d\psi\, \cos\theta\right] \ , \ \ 0 \le \psi \le 2\pi \ , \ \ 0 \le \phi \le 4\pi \ , \ \ 0 \le \theta \le \pi \ ,
\end{aligned}
\tag{3.4}
$$

with constants u_I, e_I, v_a, and $u_3 = 1/(u_1 u_2)$.

As before we denote by $f(\vec{e},\vec{v},\vec{u})$, the supergravity action evaluated on the background (3.4) and integrated over the three-sphere:

$$
f(\vec{e},\vec{v},\vec{u}) \equiv \int d\theta d\phi\, d\psi\, \sqrt{-g}\, \mathcal{L}(\vec{e},\vec{v},\vec{u}) \ . \tag{3.5}
$$

The *entropy function* $F(\vec{q},\vec{e},\vec{v},\vec{u})$ is again defined as the Legendre transform of f with respect to the charges e_I, i.e.

$$
\begin{aligned}
F(\vec{q},\vec{e},\vec{v},\vec{u}) &\equiv 2\pi \Big[e_I q^I - f(\vec{e},\vec{v},\vec{u})\Big] \\
&= 2\pi \Big[e_I q^I - \frac{\pi}{8G_5} v_1 v_2^{\frac{3}{2}} \Big(-\frac{2}{v_1} + \frac{6}{v_2} + \sum_I \frac{u_I^2 e_I^2}{2 v_1^2} + 4g^2 \sum_I u_I\Big)\Big] \ .
\end{aligned}
\tag{3.6}
$$

Note that the Chern-Simons term does not contribute to the action in the near horizon geometry (3.4). The near horizon geometry is again found by extremizing F:

$$
\frac{\partial F}{\partial e_I} \ = \ \frac{\partial F}{\partial v_a} \ = \ \frac{\partial F}{\partial u_I} \ = \ 0 \ . \tag{3.7}
$$

The first equation defines the electric charges q^I as

$$
q^I \ = \ \frac{\delta}{\delta e_I} f(\vec{e},\vec{v},\vec{u}) \ = \ \frac{\pi\, v_2^{\frac{3}{2}}}{8\, G_5\, v_1} u_I^2 e_I = -\frac{1}{16\pi G_5} \int_{S^3} X_I^2 * F^I \ . \tag{3.8}
$$

In the following we will take $G_5 = \frac{\pi}{4}$ in such a way that the charges q^I are normalized to be integers. This normalization is justified in Appendix A. The G_5 dependence can be restored by the rescaling (A.11) of the physical charges q^I.

Evaluating the entropy function at the minimum $(\vec{e}_0(\vec{q}), \vec{v}_0(\vec{q}), \vec{u}_0(\vec{q}))$ one finds the entropy of the corresponding black hole solution as a function of the electric charges $\vec{q}$.

$$
S_{BH}(\vec{q}) = F(\vec{q}, \vec{e}_0(\vec{q}), \vec{v}_0(\vec{q}), \vec{u}_0(\vec{q})) \ . \tag{3.9}
$$

3.1 The solution

In analogy to the four-dimensional case above we introduce three independent parameters μ_I to parametrize $u_{1,2}$ and v_2. The general solution of (3.7) can then be written as

$$
\begin{aligned}
u_I &= \frac{\mu_I}{(\mu_1\mu_2\mu_3)^{1/3}} \ , \qquad v_2 = (\mu_1\mu_2\mu_3)^{1/3} \ , \\
v_1 &= \frac{(\mu_1\mu_2\mu_3)^{1/3}}{4(1 + g^2 \sum_J \mu_J)} \ , \quad e_I = \frac{\sqrt{\mu_1\mu_2\mu_3(1 + g^2 \sum_{J\neq I} \mu_J)}}{2\mu_I(1 + g^2 \sum_J \mu_J)} \ , \\
q^I &= \mu_I \sqrt{1 + g^2 \textstyle\sum_{J\neq I} \mu_J} \ .
\end{aligned}
\tag{3.10}
$$

It is easy to check that equations of motion (3.3) are satisfied by (3.4), (3.10). With this solution we obtain from (3.6) for the entropy of the black hole

$$S_{\rm BH} = 2\pi\sqrt{\mu_1\mu_2\mu_3} = \frac{\pi^2 v_2^{\frac{3}{2}}}{2G_5} = \frac{1}{4G_5}\mathcal{A}_{\rm hor} , \qquad (3.11)$$

again in agreement with the Bekenstein-Hawking formula.

In lowest order of the gauge coupling we obtain the following expansion

$$\mu_I = q^I \left(1 - \tfrac{1}{2}\, g^2\, \partial_I\beta_2 + \tfrac{1}{8}\, g^4 \left(\partial_I(3\beta_1\beta_2 + 5\beta_3) - 4\beta_2\right) + \ldots\right) , \qquad (3.12)$$

in terms of the symmetric polynomials

$$\beta_1 = \sum_I q^I , \quad \beta_2 = \sum_{I<J} q^I q^J , \quad \beta_3 = q_1q_2q_3 .$$

For the entropy this implies

$$\begin{aligned} S_{\rm BH} = 2\pi\sqrt{\beta_3}\Big(1 &- \tfrac{1}{2}\, g^2\, \beta_1 + \tfrac{1}{8}\, g^4\, (3\beta_1^2 + 2\beta_2) - \tfrac{1}{16}\, g^6\, (5\beta_1^3 + 9\beta_1\beta_2 + 5\beta_3) \\ &+ \tfrac{1}{128}\, g^8\, (35\,\beta_1^4 + 116\beta_1^2\beta_2 + 20\beta_2^2 + 136\beta_1\beta_3) + \ldots\Big) . \qquad (3.13) \end{aligned}$$

Again drastic simplifications occur for $g = 0$ and for all charges equal $q^I = q$:

Ungauged theory. At $g = 0$ we have $\mu_I = q^I$ and the solution takes the explicit form

$$v_2 = 4v_1 = (q_1q_2q_3)^{\frac{1}{3}} , \qquad u_I = \frac{q^I}{(q_1q_2q_3)^{\frac{1}{3}}} , \qquad e_I = \frac{1}{2q^I}\sqrt{q_1q_2q_3} , \qquad (3.14)$$

and the black hole entropy is simply given as

$$S_{BH} = 2\pi\sqrt{q_1q_2q_3} . \qquad (3.15)$$

Equal charges $q^I = q$. In this case the above formulas reduce to

$$\begin{aligned} v_1 &= \frac{\mu}{4(1+3g^2\,\mu)} , \qquad v_2 = \mu , \qquad u_I = 1 , \\ e_I &= \frac{\sqrt{\mu + 2g^2\,\mu^2}}{2\,(1+3g^2\,\mu)} , \qquad q = \mu\sqrt{1+2g^2\,\mu} , \qquad (3.16) \end{aligned}$$

with black hole entropy

$$S_{\rm BH} = 2\pi\,\mu^{3/2} = \frac{\pi^2 v_2^{\frac{3}{2}}}{2G_5} = \frac{1}{4G_5}\mathcal{A}_{\rm hor} , \qquad (3.17)$$

expressed in terms of a single parameter μ. If instead we choose to express $S_{\rm BH}$ directly in terms of the charges q we have to invert the last equation in (3.16). A closed form for the entropy in this case is given by the more involved expression

$$S_{\rm BH} = \frac{2\sqrt{3}\,q^{3/2}}{\sqrt{\sin\phi + \sqrt{3}\cos\phi + (2/3)\,\sin 3\phi}} , \qquad \phi = \tfrac{1}{3}\arcsin(3\sqrt{3}\,q\,g^2) . \qquad (3.18)$$

4. Rotating black holes in AdS$_5$

Finally we consider rotating black holes with squashed $AdS_2 \times S^3$ near horizon geometry[2]

$$\begin{aligned}
ds^2 &= v_1 \Big(-r\,dt^2 + \frac{dr^2}{r^2} \Big) + \tfrac{1}{4} v_2 \left[\sigma_1^2 + \sigma_2^2 + v_3 (\sigma_3 - \alpha\, r\, dt)^2 \right] \,, \\
X_I &= u_I \,, \\
A^I &= -e_I\, r\, dt + p_I\, \sigma_3 \,, \qquad F^I_{0r} = e_I \,, \qquad F^I_{\psi\theta} = p_I \sin\theta \,, \\
\sigma_1^2 + \sigma_2^2 &= d\theta^2 + \sin^2\theta d\psi^2 \,, \qquad \sigma_3 = d\phi + \cos\theta d\psi \,,
\end{aligned} \tag{4.1}$$

with constants u_I, e_I, p_I, v_a, and α. The constants α, v_3 and p_I parametrize the breaking from the $SO(4)$ isometry of the non-rotating solution down to $SU(2) \times U(1)$ once the angular momentum is turned on.

The entropy function is then given by [29]

$$\begin{aligned}
F(\vec{q}, \vec{e}, \vec{v}, \vec{u}) &\equiv 2\pi \, \big(\alpha J + e_I\, \hat{q}^I - f(e_I, \alpha, v_a, u_I) \big) \\
&= 2\pi \Big[\alpha J + e_I \hat{q}^I + \frac{\pi}{3G_5} |\epsilon_{IJK}| e_I p_J p_K \\
&\quad - \frac{\pi v_1 v_2^{\frac{3}{2}} v_3^{\frac{1}{2}}}{8G_5} \Big(-\frac{2}{v_1} + \frac{8 - 2v_3}{v_2} + \frac{v_2 v_3 \alpha^2}{8 v_1^2} + \sum_I \frac{e_I^2 u_I^2}{2 v_1^2} - 8 \sum_I \frac{p_I^2 u_I^2}{v_2^2} + 4g^2 \sum_I u_I \Big) \Big] \,.
\end{aligned} \tag{4.2}$$

Notice that now also the Chern-Simons term contributes to the action. The fact that the Chern-Simons term depends explicitly on the potential A_μ rather than on the field strength $F_{\mu\nu}$ requires a slight modification of Sen's algorithm. First, the presence of the Chern-Simons term modifies the definition of the electric charge q^I. This can be easily implemented in the entropy function by the redefinition $q^I = \hat{q}^I + c^I$ with c^I chosen such that q^I are conserved quantities. Luckily the c^I induced by the Chern-Simons term are independent of e_I, u_I, and v_a such that this modification will modify neither the attractor equations nor the black hole entropy. Second, due to the presence of the Chern-Simons term, the equations of motion for A^I_ϕ

$$\begin{aligned}
0 &= \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g}\, X_I^2\, F^{\mu\phi I}) + \tfrac{1}{8} \, |\epsilon_{IJK}|\, \omega^{\mu\lambda\sigma\rho\phi}\, F^J_{\mu\lambda}\, F^K_{\sigma\rho} \,, \\
&= \frac{\alpha\, u_I^2\, e_I}{v_1^2} - \frac{16\, u_I^2\, p_I}{v_2^2} - \frac{8}{v_1\, v_2^{\frac{3}{2}}\, v_3^{\frac{1}{2}}} \, |\epsilon_{IJK}|\, e^J\, p^K \,,
\end{aligned} \tag{4.3}$$

are no longer automatically satisfied as a mere consequence of the extremization equations

$$\frac{\partial F}{\partial \alpha} = \frac{\partial F}{\partial e_I} = \frac{\partial F}{\partial u_I} = \frac{\partial F}{\partial v_a} = 0 \,. \tag{4.4}$$

Rather, equations 4.3 have to be considered in addition to the extremization equations 4.4 and determine the fluxes p_I in the ansatz (4.1).

[2]Squashing here refers to the full product, still the metric has the AdS$_2$ isometries, see [60].

The resulting solution describes the near horizon geometry of a black hole with electric charges q^I and angular momenta J given by

$$
\begin{aligned}
q^I &= \frac{\delta}{\delta e_I} f(\vec{e},\vec{v},\vec{u}) - \frac{\pi}{6\,G_5}\,|\epsilon_{IJK}|\,p_J p_K \;=\; \frac{\pi\, v_2^{\frac{3}{2}}\, v_3^{\frac{1}{2}}\, u_I^2}{8\,G_5\, v_1}\, e_I - \frac{\pi}{2\,G_5}\,|\epsilon_{IJK}|\,p_J p_K \\
&= -\frac{1}{16\pi G_5}\int_{S^3} (X_I^2 \, * F^I + \tfrac{1}{2}|\epsilon_{IJK}|\, F^J \wedge A^K) \; ,
\end{aligned}
$$

$$
J = \frac{\delta}{\delta\alpha} f(\vec{e},\vec{v},\vec{u}) \;=\; \frac{\pi\, v_2^{\frac{5}{2}}\, v_3^{\frac{3}{2}}}{32\,G_5\, v_1}\alpha = \frac{1}{16\pi G_5}\int_{S^3} *dK \; . \tag{4.5}
$$

Here $K = \frac{\partial}{\partial\phi}$ denotes the Killing vector associated with the angular rotation. The shift $c^I = -\frac{\pi}{6G_5}\,|\epsilon_{IJK}|\,p_J p_K$ has been chosen in such a way that the integrand in the definition of q^I is closed on the mass shell

$$
d(X_I^2 \, * F^I) + \tfrac{1}{2}|\epsilon_{IJK}|\, F^J \wedge F^K = 0 \; . \tag{4.6}
$$

This allow us to identify q^I with the conserved charge[3]. As we explained before neither the solution nor the entropy depends on the c^I's. In the rest of this section we describe the different subcases for which we can give explicit solutions to (4.3), (4.4).

4.1 BPS black holes

Let us first discuss the case of extremal BPS rotating black holes. These black hole solutions have been found in [44].

In this case, we can give the general solution of (4.3), 4.4 again in terms of three independent parameters μ_I and their symmetric polynomials

$$
\gamma_1 = \sum_I \mu_I \; , \quad \gamma_2 = \sum_{I<J} \mu_I \mu_J \; , \quad \gamma_3 = \mu_1\mu_2\mu_3 \; ,
$$

as follows

$$
\begin{aligned}
u_I &= \frac{\mu_I}{\gamma_3^{1/3}} \; , \qquad v_1 = \frac{\gamma_3^{1/3}}{4(1+g^2\,\gamma_1)} \; , \quad v_2 = \gamma_3^{1/3} \; , \\
v_3 &= 1 + g^2\,\gamma_1 - \frac{g^2\,\gamma_2^2}{4\gamma_3} \; , \qquad \alpha = \frac{g\,\gamma_2}{(1+g^2\,\gamma_1)\sqrt{4\gamma_3\,(1+g^2\,\gamma_1) - g^2\,\gamma_2^2}} \; , \\
e_I &= \frac{\sqrt{4\gamma_3\,(1+g^2\,\gamma_1) - g^2\,\gamma_2^2}}{4\mu_I\,(1+g^2\,\gamma_1)} \; , \qquad p_I = \tfrac{1}{4}\, g\,(\gamma_1 - \mu_I) - \frac{g\,\gamma_3}{4\mu_I^2} \; , \\
q^I &= \mu_I + \tfrac{1}{2}\, g^2\, \mu_I(\gamma_1 - \mu_I) - \frac{g^2\,\gamma_3}{2\mu_I} \qquad J = \frac{g\,\gamma_2\,(4\gamma_3\,(1+g^2\,\gamma_1) - g^2\,\gamma_2^2)}{16\gamma_3}
\end{aligned} \tag{4.7}
$$

[3] J.F.M. thanks L.Alvarez-Gaume and C.N. Pope for useful discussions on this point.

Plugging this into 4.2 we obtain for the entropy

$$S_{\rm BH} \;=\; 2\pi\,\sqrt{\gamma_3\,(1+g^2\,\gamma_1)-\tfrac14\,g^2\,\gamma_2^2} \;=\; \frac{\pi^2 v_2^{\frac32}\,v_3^{\frac12}}{2G_5} \;=\; \frac{1}{4G_5}\mathcal{A}_{\rm hor}\;, \tag{4.8}$$

reproducing the result of [44]. In order to compare the results it is helpful to note that the parametrization of the squashed $AdS_2\times S^3$ near horizon geometry given in [44]

$$\begin{aligned} ds^2_{\rm BPS} &= -f^2\,dT^2+2f^2\,w\,dT\sigma_3+f^{-1}\,b^{-1}\,dR^2+\tfrac14\,R^2\,f^{-1}\,(\sigma_1^2+\sigma_2^2+c\,\sigma_3^2)\;,\\ f &= R^2\gamma_3^{-\frac13}\;,\quad w=-\frac{\gamma_2 g}{4R^2}\;,\quad b=1+g^2\gamma_1\;,\quad c=1+g^2\gamma_1-\frac{g^2\gamma_2^2}{4\gamma_3}\;, \end{aligned} \tag{4.9}$$

translates into the standard form (4.1) with

$$r=R^2\;,\quad dt=\frac{2\,b}{\sqrt{\gamma_3\,c}}\,dT\;,\quad v_1=\frac{\gamma_3^{\frac13}}{4b}\;,\quad v_2=\gamma_3^{-\frac13}\;,\quad v_3=c\;,\quad \alpha=\frac{\gamma_2\,g}{2\,b\sqrt{\gamma_3\,c}}\;.$$

in agreement with (4.7).

4.2 Non-extremal black holes

These considerations can be extended to non-extremal black holes. For simplicity we focus on the case of equal charges $q^I=q$. The general solution of equations (4.3), 4.4 can then be expressed in terms of two independent parameters μ, $\omega>1$ as

$$\begin{aligned} u_I &= 1\;,\qquad v_1=\frac{\mu}{4(1+3g^2\mu)}\;,\qquad v_2=\mu\;,\quad v_3=\mu^{-3}\Delta_s^2\;, \\ e_I &= \frac{\Delta_s}{2\mu(\omega-1)(1+3g^2\mu)}\;,\qquad p_I=\frac{\Delta_\alpha}{2\mu(1+w)}\;,\qquad \alpha=\frac{\Delta_\alpha}{\Delta_s(1+3g^2\mu)}\;,\\ J &= \tfrac12\,\mu^{-3}\,\Delta_\alpha\Delta_s^2\;,\qquad q^I=\frac{2\mu}{\omega}+2\,g^2\,\mu^2\frac{(\omega-1)}{\omega^2}\;, \end{aligned} \tag{4.10}$$

with

$$\begin{aligned} \Delta_\alpha &= \frac{\mu(\omega+1)}{\omega^2}\,\sqrt{2\mu\omega(\omega-2)+4g^2\mu^2(\omega^2-2\omega+1)}\;,\\ \Delta_s &= \frac{\mu(\omega-1)}{\omega^2}\,\sqrt{2\mu\omega(\omega+2)+2g^2\mu^2(\omega^2+2\omega-2)}\;. \end{aligned} \tag{4.11}$$

Plugging this into 4.2, we find for the entropy

$$S_{\rm BH}=2\pi\Delta_s \;=\; \frac{\pi^2 v_2^{\frac32}\,v_3^{\frac12}}{2G_5} = \frac{1}{4G_5}\mathcal{A}_{\rm hor}\;. \tag{4.12}$$

It is interesting to note that although for a generic choice of the parameters μ, ω the black hole solution found here is non-supersymmetric, the charges can be chosen in such a way that the BPS bound is saturated. More precisely, for the particular value $\omega=2$ the formulas obtained here reduce to

$$
\begin{aligned}
& u_I = 1 \,, \quad e_I = \frac{\sqrt{\mu}\sqrt{4+3g^2\,\mu}}{4\,(1+3g^2\,\mu)} \,, \quad p_I = \tfrac{1}{4}\,g\,\mu \,, \quad q^I = \mu + \tfrac{1}{2}\,g^2\,\mu^2 \,, \\
& v_1 = \frac{\mu}{4(1+3g^2\,\mu)} \,, \quad v_2 = \mu \,, \quad v_3 = 1 + \tfrac{3}{4}\,g^2\,\mu \,, \quad \alpha = \frac{3g\,\sqrt{\mu}}{(1+3g^2\,\mu)\sqrt{4+3g^2\mu)}} \,, \\
& J = \frac{3}{16}\,g\,\mu^2\,(4+3g^2\mu) \,, \qquad S_{\rm BH} = 2\pi\,\mu^{3/2}\,\sqrt{1+\tfrac{3}{4}\,g^2\,\mu} \,, \qquad (4.13)
\end{aligned}
$$

that agrees with the general BPS solution 4.7 after taking all charges equal $\mu_i = \mu$.

Another interesting limit of the solution (4.10) is the unrotating case studied in last section. This is given by setting

$$\omega = 1 + \frac{1}{\sqrt{1+2g^2\mu}} \,. \qquad (4.14)$$

Indeed it is straightforward to check that at this value the above formulas reduce to (3.16).

5. Higher derivative terms

Finally we consider asymptotically Anti-de Sitter black hole horizons in higher derivative gravity. In contrast to the case of Poincaré supergravities, higher derivative couplings in gauged supergravities were rarely studied in the string literature. Awaiting more realistic Lagrangians here we illustrate the entropy formalism in an archetype toy example: the Einstein-Maxwell system in presence of a Gauss-Bonnet term and a cosmological constant

$$S = \frac{1}{16\pi G_d}\int d^dx\sqrt{-g}\left(R - \tfrac{1}{4}F^2 + \Lambda + a\,\mathcal{L}_{\rm GB}\right) \,, \qquad (5.1)$$

with the Gauss-Bonnet term

$$\mathcal{L}_{GB} = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \,. \qquad (5.2)$$

The parameter a measures the deviation from Einstein gravity and it depends on the particular string model under consideration.[4]

The equations of motion following from (5.1) are:

$$
\begin{aligned}
& R_{\mu\nu} - \tfrac{1}{2}F_{\mu\sigma}F_\nu{}^\sigma + a\,\frac{\delta\mathcal{L}_{\rm GB}}{\delta g^{\mu\nu}} - \tfrac{1}{2}g_{\mu\nu}\left(R - \tfrac{1}{4}F^2 + \Lambda + a\,\mathcal{L}_{\rm GB}\right) = 0 \,, \\
& \partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0 \,, \qquad (5.3)
\end{aligned}
$$

with

$$\frac{\delta\mathcal{L}_{\rm GB}}{\delta g^{\mu\nu}} \;=\; 2(R_{\mu\sigma\rho\delta}\,R_\nu{}^{\sigma\rho\delta} - 2\,R^{\rho\sigma}\,R_{\mu\rho\nu\sigma} - 2\,R_\mu^\sigma\,R_{\nu\sigma} + R\,R_{\mu\nu}) \,, \qquad (5.4)$$

[4] See [61 – 63] for an analysis of the boundary terms needed by the regularization of the action for Einstein-Gauss-Bonnet-AdS gravity.

up to total derivatives. We look for $AdS_2 \times S^{d-2}$ near horizon geometries:

$$ds^2 = v_1 \left(-r^2\, dt^2 + \frac{dr^2}{r^2} \right) + v_2\, d\Omega_{d-2} \,, \qquad F_{0r} = e \,. \tag{5.5}$$

The extremization equations of the entropy function can now be explicitly solved in the different space-time dimensions.

d=4. In four dimensions, evaluating the entropy function for this system yields

$$F(q,e,\vec{v}) \equiv 2\pi \left[eq - \frac{v_1 v_2}{4G_4} \left(-\frac{2}{v_1} + \frac{2}{v_2} - \frac{8a}{v_1 v_2} + \frac{e^2}{2v_1^2} + \Lambda \right) \right] . \tag{5.6}$$

The extremum of $F(q,e,\vec{v})$ (for a fixed q) can be conveniently parametrized in terms of v_2:

$$v_1 = \frac{v_2}{1+v_2\Lambda} \,, \qquad e = \sqrt{\frac{2v_2(2+v_2\Lambda)}{(1+v_2\Lambda)}} \,, \qquad q = \frac{1}{2G_4}\sqrt{v_2(1+\tfrac{1}{2}v_2\Lambda)} \,. \tag{5.7}$$

Plugging (5.7) into the entropy function (5.6) one finds the black hole entropy

$$S_{\rm BH} = \frac{\pi}{G_4}\,(v_2 + 4a) \,. \tag{5.8}$$

The a-term gives the deviation from the area law due to the Gauss-Bonnet term. Interestingly, the presence of the Gauss-Bonnet term in $d = 4$ does not modify the near horizon solution but only the black hole entropy. This is consistent with the fact that in $d = 4$ the a-dependent term in the equations of motion (5.3) cancels once evaluated on $AdS_2 \times S^2$. In $d = 5$ this will be different as we shall see.

d=5. In five dimensions the entropy function is given by

$$F(q,e,\vec{v}) \equiv 2\pi \left[eq - \frac{\pi v_1 v_2^{\frac{3}{2}}}{8G_5} \left(-\frac{2}{v_1} + \frac{6}{v_2} - \frac{24a}{v_1 v_2} + \frac{e^2}{2v_1^2} + \Lambda \right) \right] . \tag{5.9}$$

The extremum of $F(q,e,\vec{v})$ (for a fixed q) can be conveniently parametrized in terms of the sphere radius v_2:

$$v_1 = \frac{v_2+4a}{4-v_2\Lambda} \,, \quad e = \left(\frac{v_2+4a}{4-v_2\Lambda} \right) \sqrt{12\, v_2^{-1} - 2\,\Lambda} \,, \quad q = \frac{\pi v_2}{4G_5}\sqrt{3-\tfrac{1}{2}v_2\Lambda} \,. \tag{5.10}$$

Plugging (5.10) into the entropy function (5.9) one finds the black hole entropy

$$S_{\rm BH} = \frac{\pi^2 v_2^{\frac{1}{2}}}{2G_5}\,(v_2 + 12a) \,. \tag{5.11}$$

The a-dependent term represents the deviation from the area law due to the Gauss-Bonnet term.

6. Conclusions

In this paper we applied the entropy formalism to the case of gauged supergravities which admit asymptotically AdS electrically charged black holes with $AdS_2 \times S^{d-2}$ horizons. Using Sen's algorithm we have determined the fixed near-horizon geometries for four and five-dimensional static black holes, for rotating five-dimensional black holes and finally for AdS black holes with higher derivative corrections of Gauss-Bonnet type. In each case we find horizons with fixed scalars, AdS and sphere radii, determined entirely in terms of the gauge coupling, the black hole electric charges and the angular momentum.

The explicit dependence on the gauge potential via the Chern-Simons term in the five-dimensional gauged supergravity requires a slight modification of the entropy function algorithm. We have illustrated this in the case of five-dimensional rotating black holes. Once the black hole rotates, magnetic fluxes p_I should be turned on and the Chern-Simons term starts contributing to the action. The inclusion of this term leads to a redefinition of the black hole electric charge $q^I \to q^I + c^I$ with c^I depending only on the magnetic fluxes and not on the metric or scalar fields. This implies in particular that neither the attractor equations nor the entropy depends on c^I and therefore c^I can be adjusted to account for the Chern-Simons correction to the electric charge. The fluxes p_I are determined by an extra constraint coming from the gauge field equations of motion to be imposed in addition to the extremization conditions of the entropy function. Nicely, this leads again to a family of black hole near horizon solutions parametrized only by the black hole electric charges and the angular momentum.

In the case of Einstein gravity, the near horizon geometries derived here can be recovered by considering the zero temperature limit of the general black hole solutions [39 – 50]. In this limit one finds a single horizon with $AdS_2 \times S^{d-2}$ topology. We stress the fact that in the gauged theory, zero temperature black holes are not necessarily supersymmetric. A non-BPS black hole solution is known to be classified by its charges (electric charge, angular momentum, etc.) and its mass. The condition of zero temperature relates the black hole mass to its charges. This implies that there is a unique black hole solution with $AdS_2 \times S^{d-2}$ horizon for a given choice of the charges. This is precisely the result coming from extremizing the entropy function. The precise matching between our solutions and the $T \to 0$ limit of the general non-extremal black hole solutions is shown explicitly in appendix B for static black holes and in section 4.1 for the five-dimensional BPS case [44].

It is tempting to speculate about the generalization of the expressions for the entropy (2.14), (3.13) to the full $\mathcal{N} = 8$ theories. For the ungauged case $g = 0$ it is well known that the first term in the expansions is replaced by the quartic and cubic invariants of the global symmetry groups E_7 and E_6, respectively [64]. The gauging of the theories is most conveniently described in terms of an embedding tensor which parametrizes the deformation in order g and comes in a particular representation of the global symmetry groups [65]. This suggests that e.g. the second term in the expansion (2.14) will be replaced by an E_7 invariant built from six charges and two embedding tensors. Indeed, there is a single nontrivial E_7 invariant combination of these representations which might thus generalize the expansion (2.14) to lowest orders.

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It would be nice to explore the implications of our results to gauge/gravity holographic correspondences. In particular, the AdS_5 entropy formula provide explicit predictions on the partition function of gauge invariant operators in $\mathcal{N}=4$ SYM. In addition the $AdS_2 \times S^{d-2}$ solutions found here can be used as starting points of new holographic relations between quantum mechanical systems living on the AdS_2 boundary and the gravity physics near the horizon.

We hope to come back to some of these issues in the near future.

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A. Physical charge units

In this appendix we explain the normalization of physical charges adopted in the text. Electric charge units do not depend on the coupling constant g, therefore we can restrict ourselves to the ungauged limit $g = 0$. The five and four-dimensional supergravities studied here can be embedded into compactifications of type II supergravities on T^5 and T^6 respectively. The black hole solutions in this limit reduce to the well known 3- and 4-charge black hole solutions of the maximal supergravities. Here we normalize our charges in such a way as to match the electric charge units coming from black holes built out of branes in string theory. The formulas in this appendix follow the notations and conventions in [66]. We refer the reader to this reference for further details and a complete list of references on the subject.

Newton constant.

$$G_d = \frac{G_{10}}{(2\pi)^{10-d} V_{10-d}} , \qquad G_{10} = 8\pi^6 g_s^2 \ell_s^8 , \tag{A.1}$$

with string length $\ell_s = \sqrt{\alpha'}$, string coupling constant g_s, and the volume V_{10-d} of the compactification manifold.

4-charge black hole. The Einstein metric of a 4-charge black hole in $d = 4$ dimensions can be written as

$$\begin{aligned} ds^2 &= -(H_1 H_2 H_3 H_4)^{-\frac{1}{2}}\, dT^2 + (H_1 H_2 H_3 H_4)^{\frac{1}{2}}\, (dr^2 + r^2 d\Omega_2) , \\ H_i &= 1 + \frac{c_i N_i}{r} , \end{aligned} \tag{A.2}$$

with integers N_i counting the number of brane constituents and some constants c_i parametrizing the brane tension. In the near horizon $r \to 0$, the black hole geometry becomes

$$ds^2 = -(cN_1N_2N_3N_4)^{-\frac{1}{2}}\, r^2\, dT^2 + (cN_1N_2N_3N_4)^{\frac{1}{2}}\, \frac{dr^2}{r^2} + (cN_1N_2N_3N_4)^{\frac{1}{2}} d\Omega_2 , \tag{A.3}$$

with

$$c = c_1c_2c_3c_4 = \frac{g_s^4 \ell_s^{16}}{16V_6^2} = 4G_4^2 \ . \tag{A.4}$$

Notice that although c_i depends on the type of brane constituent and on the string model, c is a U-duality invariant quantity that depends only on G_4.

After a rescaling of dT the metric (A.3) can be put into our standard $AdS_2 \times S^2$ form (2.4) with

$$v_1 = v_2 = \sqrt{cN_1N_2N_3N_4} = 2G_4\sqrt{N_1N_2N_3N_4} \ . \tag{A.5}$$

Taking $G_4 = \frac{1}{8}$ and comparing (A.5) with (2.15), one finds agreement with the identification $q_i = N_i$, i.e. the q_i are integers. It is important to note that G_4 can be reabsorbed by a simultaneous rescaling of q_i and $S_{\rm BH}$. Therefore the G_4 dependence in the main text can be restored by sending

$$q_i \to (8G_4)\, q_i \ , \qquad S_{\rm BH} \to (8G_4)\, S_{\rm BH} \ . \tag{A.6}$$

Clearly, the q_i's defined in this way will not be integers.

3-charge black hole. The Einstein metric of a 3-charge black hole in $d = 5$ dimensions can be written as

$$\begin{aligned} ds^2 &= -(H_1H_2H_3)^{-\frac{2}{3}}\, dT^2 + (H_1H_2H_3)^{\frac{1}{3}}\, (dR^2 + R^2 d\Omega_3) \ , \\ H_i &= 1 + \frac{c_iN_i}{R^2} \ . \end{aligned} \tag{A.7}$$

In the near horizon $r = R^2 \to 0$, the black hole geometry becomes

$$ds^2 = -(cN_1N_2N_3)^{-\frac{1}{2}}\, r^2\, dT^2 + (cN_1N_2N_3)^{\frac{1}{2}}\, \frac{dr^2}{4r^2} + (cN_1N_2N_3)^{\frac{1}{2}} d\Omega_3 \ , \tag{A.8}$$

with

$$c = c_1c_2c_3 = \frac{g_s^4 \ell_s^{16}}{V_6^2} = \left(\frac{4G_5}{\pi}\right)^2 . \tag{A.9}$$

Again c is a U-duality invariant quantity depending only on G_5.

After a rescaling of dT the metric (A.8) can be put into the standard $AdS_2 \times S^3$ form (3.4) with

$$v_2 = 4v_1 = (cN_1N_2N_3)^{\frac{1}{3}} = \left(\frac{4G_5}{\pi}\right)^{\frac{2}{3}} (N_1N_2N_3)^{\frac{1}{3}} \ . \tag{A.10}$$

Taking $G_5 = \frac{\pi}{4}$ and comparing (A.10) with (3.14), one finds agreement with the identification $q_i = N_i$, i.e. the q_i are integers.

It is important to note that G_5 can be reabsorbed by a simultaneous rescaling of q_i and $S_{\rm BH}$. Therefore the G_5 dependence in the main text can be restored by sending

$$q_i \to \left(\frac{4G_5}{\pi}\right) q_i \ , \qquad S_{\rm BH} \to \left(\frac{4G_5}{\pi}\right) S_{\rm BH} \ . \tag{A.11}$$

Clearly the q_i's defined in this way will not be integers.

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B. Black holes at $T = 0$

In this Appendix we show that the $AdS_2 \times S^{d-2}$ geometries derived in the text agree with those coming by taking the zero temperature limit of the most general non-extremal black hole solutions in $d = 4, 5$ dimensions. For simplicity we focus on the static case. We refer the reader to [51] for details and references on the AdS black hole solutions quoted in this Appendix.

$d = 4$ case. The general non-extremal and static asymptotically AdS black hole solution of $U(1)^4$ gauged supergravity in $d = 4$ can be written as[5]:

$$\begin{aligned} ds_4^2 &= H^{-2}\, f\, dt^2 + H^2\, (f^{-1}\, dr^2 + r^2\, d\Omega_2) \ , \\ X_I &= \frac{H_I}{H} \ , \qquad\qquad F^I \ = \ dH_I^{-1} \coth\beta_i dt \ , \end{aligned} \tag{B.1}$$

with

$$f = 1 - \frac{m}{r} + 4\, g^2\, r^2\, H^4 \ , \qquad H^4 = H_1 H_2 H_3 H_4 \ , \qquad H_I = 1 + \frac{m \sinh^2\beta_i}{r} \ . \tag{B.2}$$

The parameters β_i and m parametrize the electric charges and mass of the black hole. For a generic choice of m the black hole has two horizons at $r_\pm$ given by the zeros of f. The two horizons coincide when $r_0 = r_+ = r_-$, i.e. when both f and its first derivative vanish at $r = r_0$:

$$f(r_0) = f'(r_0) = 0 \ . \tag{B.3}$$

Denoting

$$\begin{aligned} &\tfrac{1}{2}\mu_I = r_0\, H_I(r_0) \ , \\ &\gamma_1 = \sum_I \mu_I \ , \quad \gamma_2 = \sum_{I<J} \mu_I \mu_J \ , \quad \gamma_3 = \sum_{I<J<K} \mu_I \mu_J \mu_K \ , \quad \gamma_4 = \mu_1\mu_2\mu_3\mu_4 \ , \end{aligned} \tag{B.4}$$

equations (B.3) can be solved for m and r_0 in terms of μ_I:

$$m = g\, \sqrt{\gamma_4 + \tfrac{1}{4}\, g^2 \gamma_3^2} \ , \qquad r_0 = \tfrac{1}{2}\, m - \tfrac{1}{4}\, g^2\, \gamma_3 \ . \tag{B.5}$$

The temperature of the black hole is zero for this choice and the horizon geometry takes the $AdS_2 \times S^2$ form with

$$v_1 = \tfrac{1}{2}\, H(r_0)^2\, f''(r_0)^{-1} = \frac{\tfrac{1}{4}\sqrt{\gamma_4}}{1 + g^2\, \gamma_2} \ , \qquad v_2 \ = \ r_0^2\, H(r_0)^2 = \tfrac{1}{4}\sqrt{\gamma_4} \ , \tag{B.6}$$

in precise agreement with (2.11).

[5] The X_I's here are the inverse of the X_i's used in [51]

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$d = 5$ **case.** The general non-extremal and static asymptotically AdS black hole solution of $U(1)^3$ gauged supergravity in $d = 5$ dimensions can be written as[6]:

$$\begin{aligned} ds_4^2 &= H^{-2} f\, dt^2 + H\,(f^{-1}\, dr^2 + r^2\, d\Omega_2)\,, \\ X_I &= \frac{H_I}{H}\,, \qquad\qquad F^I \;=\; dH_I^{-1} \coth\beta_i dt\,, \end{aligned} \tag{B.7}$$

with

$$f = 1 - \frac{m}{r^2} + g^2\, r^2\, H^3\,, \qquad H^3 = H_1 H_2 H_3\,, \qquad H_I = 1 + \frac{m \sinh^2\beta_i}{r^2}\,. \tag{B.8}$$

The parameters β_i and m parametrize the electric charges and mass of the black hole. For a generic choice of m the black hole has two horizons at $r_\pm$ given by the two positive zeros of f. The two horizons coincide $r_0 = r_\pm$ when parameters are chosen such that both f and its first derivative vanish at the horizon:

$$f(r_0) = f'(r_0) = 0\,. \tag{B.9}$$

Denoting

$$\mu_I = r_0^2\, H_I(r_0)\,, \qquad \gamma_1 = \sum_I \mu_I\,, \quad \gamma_2 = \sum_{I<J} \mu_I \mu_J\,, \quad \gamma_3 = \mu_1\mu_2\mu_3\,,$$

equations (B.9) can be solved for m and r_0 in terms of μ_I:

$$m = g\,\sqrt{4\gamma_3 + g^2\gamma_2^2}\,, \qquad r_0^2 = \tfrac{1}{2}m - \tfrac{1}{2}\, g^2\, \gamma_2\,. \tag{B.10}$$

The temperature of the black hole is zero for this choice and the horizon geometry takes the $AdS_2 \times S^3$ form with:

$$v_1 \;=\; \tfrac{1}{2}\, r_0^4\, H(r_0)\, f''(r_0)^{-1} = \frac{\frac{1}{4}\gamma_3^{\frac{1}{3}}}{1 + g^2\,\gamma_1}\,, \qquad v_2 \;=\; r_0^2\, H(r_0) = \gamma_3^{\frac{1}{3}}\,, \tag{B.11}$$

in agreement with (3.10).

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S. Ferrara, R. Kallosh and A. Strominger, *$N = 2$ extremal black holes*, *Phys. Rev.* **D 52** (1995) 5412 [hep-th/9508072].

[3] A. Strominger, *Macroscopic entropy of $N = 2$ extremal black holes*, *Phys. Lett.* **B 383** (1996) 39 [hep-th/9602111].

[4] S. Ferrara and R. Kallosh, *Supersymmetry and attractors*, *Phys. Rev.* **D 54** (1996) 1514 [hep-th/9602136].

[6] The X_I's here are the inverse of the X_i's used in [51]

[5] S. Ferrara and R. Kallosh, *Universality of supersymmetric attractors*, *Phys. Rev.* **D 54** (1996) 1525 [hep-th/9603090].

[6] K. Goldstein, N. Iizuka, R.P. Jena and S.P. Trivedi, *Non-supersymmetric attractors*, *Phys. Rev.* **D 72** (2005) 124021 [hep-th/0507096].

[7] R. Kallosh, *New attractors*, *JHEP* **12** (2005) 022 [hep-th/0510024].

[8] P.K. Tripathy and S.P. Trivedi, *Non-supersymmetric attractors in string theory*, *JHEP* **03** (2006) 022 [hep-th/0511117].

[9] A. Giryavets, *New attractors and area codes*, *JHEP* **03** (2006) 020 [hep-th/0511215].

[10] K. Goldstein, R.P. Jena, G. Mandal and S.P. Trivedi, *A c-function for non-supersymmetric attractors*, *JHEP* **02** (2006) 053 [hep-th/0512138].

[11] R. Kallosh, N. Sivanandam and M. Soroush, *The non-BPS black hole attractor equation*, *JHEP* **03** (2006) 060 [hep-th/0602005].

[12] B. Chandrasekhar, S. Parvizi, A. Tavanfar and H. Yavartanoo, *Non-supersymmetric attractors in R^2 gravities*, *JHEP* **08** (2006) 004 [hep-th/0602022].

[13] S. Bellucci, S. Ferrara and A. Marrani, *On some properties of the attractor equations*, *Phys. Lett.* **B 635** (2006) 172 [hep-th/0602161].

[14] R. Kallosh, *From BPS to non-BPS black holes canonically*, hep-th/0603003.

[15] S. Ferrara and R. Kallosh, *On $N = 8$ attractors*, *Phys. Rev.* **D 73** (2006) 125005 [hep-th/0603247].

[16] S. Ferrara and M. Günaydin, *Orbits and attractors for $N = 2$ Maxwell-Einstein supergravity theories in five dimensions*, hep-th/0606108.

[17] S. Bellucci, S. Ferrara, M. Günaydin and A. Marrani, *Charge orbits of symmetric special geometries and attractors*, hep-th/0606209.

[18] S. Ferrara, E.G. Gimon and R. Kallosh, *Magic supergravities, $N = 8$ and black hole composites*, hep-th/0606211.

[19] R. Kallosh, N. Sivanandam and M. Soroush, *Exact attractive non-BPS stu black holes*, *Phys. Rev.* **D 74** (2006) 065008 [hep-th/0606263].

[20] A. Sen, *Black hole entropy function and the attractor mechanism in higher derivative gravity*, *JHEP* **09** (2005) 038 [hep-th/0506177].

[21] A. Sen, *Entropy function for heterotic black holes*, *JHEP* **03** (2006) 008 [hep-th/0508042].

[22] M. Alishahiha and H. Ebrahim, *Non-supersymmetric attractors and entropy function*, *JHEP* **03** (2006) 003 [hep-th/0601016].

[23] B. Sahoo and A. Sen, *Higher derivative corrections to non-supersymmetric extremal black holes in $N = 2$ supergravity*, hep-th/0603149.

[24] G.L. Cardoso, D. Lüst and J. Perz, *Entropy maximization in the presence of higher-curvature interactions*, *JHEP* **05** (2006) 028 [hep-th/0603211].

[25] B. Chandrasekhar, *Born-Infeld corrections to the entropy function of heterotic black holes*, hep-th/0604028.

JHEP10(2006)074

[26] A. Ghodsi, *R^4 corrections to D1D5P black hole entropy from entropy function formalism*, hep-th/0604106.

[27] M. Alishahiha and H. Ebrahim, *New attractor, entropy function and black hole partition function*, hep-th/0605279.

[28] R.-G. Cai and D.-W. Pang, *Entropy function for 4-charge extremal black holes in type-IIA superstring theory*, *Phys. Rev.* **D 74** (2006) 064031 [hep-th/0606098].

[29] D. Astefanesei, K. Goldstein, R.P. Jena, A. Sen and S.P. Trivedi, *Rotating attractors*, hep-th/0606244.

[30] G.L. Cardoso, V. Grass, D. Lüst and J. Perz, *Extremal non-BPS black holes and entropy extremization*, *JHEP* **09** (2006) 078 [hep-th/0607202].

[31] S. Kar and S. Majumdar, *Noncommutative D3-brane, black holes and attractor mechanism*, *Phys. Rev.* **D 74** (2006) 066003 [hep-th/0606026].

[32] S. Kar, *Noncommutative brane-world, (anti) de Sitter vacua and extra dimensions*, hep-th/0608018.

[33] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[34] B. Sundborg, *The hagedorn transition, deconfinement and $N = 4$ SYM theory*, *Nucl. Phys.* **B 573** (2000) 349 [hep-th/9908001].

[35] O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, *The hagedorn/deconfinement phase transition in weakly coupled large-N gauge theories*, *Adv. Theor. Math. Phys.* **8** (2004) 603 [hep-th/0310285].

[36] J. Kinney, J.M. Maldacena, S. Minwalla and S. Raju, *An index for 4 dimensional super conformal theories*, hep-th/0510251.

[37] L. Alvarez-Gaumé, C. Gomez, H. Liu and S. Wadia, *Finite temperature effective action, AdS_5 black holes and 1/n expansion*, *Phys. Rev.* **D 71** (2005) 124023 [hep-th/0502227].

[38] L. Alvarez-Gaumé, P. Basu, M. Marino and S.R. Wadia, *Blackhole/string transition for the small Schwarzschild blackhole of $AdS_5 \times S^5$ and critical unitary matrix models*, hep-th/0605041.

[39] K. Behrndt, M. Cvetič and W.A. Sabra, *Non-extreme black holes of five dimensional $N = 2$ AdS supergravity*, *Nucl. Phys.* **B 553** (1999) 317 [hep-th/9810227].

[40] M. Cvetič et al., *Embedding AdS black holes in ten and eleven dimensions*, *Nucl. Phys.* **B 558** (1999) 96 [hep-th/9903214].

[41] M.J. Duff and J.T. Liu, *Anti-de Sitter black holes in gauged $N = 8$ supergravity*, *Nucl. Phys.* **B 554** (1999) 237 [hep-th/9901149].

[42] W.A. Sabra, *Anti-de Sitter BPS black holes in $N = 2$ gauged supergravity*, *Phys. Lett.* **B 458** (1999) 36 [hep-th/9903143].

[43] J.B. Gutowski and H.S. Reall, *Supersymmetric AdS_5 black holes*, *JHEP* **02** (2004) 006 [hep-th/0401042].

[44] J.B. Gutowski and H.S. Reall, *General supersymmetric AdS_5 black holes*, *JHEP* **04** (2004) 048 [hep-th/0401129].

JHEP10(2006)074

[45] M. Cvetič, H. Lu and C.N. Pope, *Charged kerr-de Sitter black holes in five dimensions*, *Phys. Lett.* **B 598** (2004) 273 [hep-th/0406196].

[46] M. Cvetič, H. Lu and C.N. Pope, *Charged rotating black holes in five dimensional $U(1)^3$ gauged $N = 2$ supergravity*, *Phys. Rev.* **D 70** (2004) 081502 [hep-th/0407058].

[47] M. Cvetič, G.W. Gibbons, H. Lu and C.N. Pope, *Rotating black holes in gauged supergravities: thermodynamics, supersymmetric limits, topological solitons and time machines*, hep-th/0504080.

[48] Z.W. Chong, M. Cvetič, H. Lu and C.N. Pope, *Five-dimensional gauged supergravity black holes with independent rotation parameters*, *Phys. Rev.* **D 72** (2005) 041901 [hep-th/0505112].

[49] Z.W. Chong, M. Cvetič, H. Lu and C.N. Pope, *General non-extremal rotating black holes in minimal five-dimensional gauged supergravity*, *Phys. Rev. Lett.* **95** (2005) 161301 [hep-th/0506029].

[50] H.K. Kunduri, J. Lucietti and H.S. Reall, *Supersymmetric multi-charge AdS_5 black holes*, *JHEP* **04** (2006) 036 [hep-th/0601156].

[51] M.J. Duff, *Lectures on branes, black holes and anti-de Sitter space*, hep-th/9912164.

[52] G. Lopes Cardoso, B. de Wit and T. Mohaupt, *Corrections to macroscopic supersymmetric black-hole entropy*, *Phys. Lett.* **B 451** (1999) 309 [hep-th/9812082].

[53] G. Lopes Cardoso, B. de Wit and T. Mohaupt, *Macroscopic entropy formulae and non-holomorphic corrections for supersymmetric black holes*, *Nucl. Phys.* **B 567** (2000) 87 [hep-th/9906094].

[54] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, *Stationary BPS solutions in $N = 2$ supergravity with R^2 interactions*, *JHEP* **12** (2000) 019 [hep-th/0009234].

[55] A. Dabholkar, *Exact counting of black hole microstates*, *Phys. Rev. Lett.* **94** (2005) 241301 [hep-th/0409148].

[56] M. Cvetič, S. Nojiri and S.D. Odintsov, *Black hole thermodynamics and negative entropy in desitter and anti-desitter Einstein-Gauss-Bonnet gravity*, *Nucl. Phys.* **B 628** (2002) 295 [hep-th/0112045].

[57] S. Dutta and R. Gopakumar, *On euclidean and noetherian entropies in AdS space*, *Phys. Rev.* **D 74** (2006) 044007 [hep-th/0604070].

[58] B. de Wit and H. Nicolai, *$N = 8$ supergravity*, *Nucl. Phys.* **B 208** (1982) 323.

[59] M. Günaydin, L.J. Romans and N.P. Warner, *Compact and noncompact gauged supergravity theories in five-dimensions*, *Nucl. Phys.* **B 272** (1986) 598.

[60] P. Davis, H.K. Kunduri and J. Lucietti, *Special symmetries of the charged kerr-AdS black hole of $D = 5$ minimal gauged supergravity*, *Phys. Lett.* **B 628** (2005) 275 [hep-th/0508169].

[61] R. Aros, M. Contreras, R. Olea, R. Troncoso and J. Zanelli, *Conserved charges for gravity with locally AdS asymptotics*, *Phys. Rev. Lett.* **84** (2000) 1647 [gr-qc/9909015].

[62] R. Olea, *Mass, angular momentum and thermodynamics in four-dimensional kerr-AdS black holes*, *JHEP* **06** (2005) 023 [hep-th/0504233].

[63] G. Kofinas and R. Olea, *Vacuum energy in Einstein-Gauss-Bonnet AdS gravity*, hep-th/0606253.

[64] R. Kallosh and B. Kol, *E_7 symmetric area of the black hole horizon*, *Phys. Rev.* **D 53** (1996) 5344 [hep-th/9602014].

[65] B. de Wit, H. Samtleben and M. Trigiante, *On lagrangians and gaugings of maximal supergravities*, *Nucl. Phys.* **B 655** (2003) 93 [hep-th/0212239].

[66] A.W. Peet, *TASI lectures on black holes in string theory*, hep-th/0008241.